

MIDPOINT OF A LINE

The midpoint of a line segment is the point exactly halfway between the endpoints. Because finding an ordered pair requires movement in two directions, to calculate the midpoint between two ordered pairs, you need to go halfway each of the x-coordinates and the y-coordinates.

The midpoint of two points is the point located halfway between them. We can think of finding a midpoint as finding the average of the x coordinates and the averages of the y coordinates. Therefore, for points (x_1, y_1) and (x_2, y_2) , the midpoint formula is:

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

EXAMPLE

The midpoint of $A(5, -8)$ and $B(-1, 2)$ is:

$$\left(\frac{5 + (-1)}{2}, \frac{-8 + 2}{2} \right) = \left(\frac{4}{2}, \frac{-6}{2} \right) = (2, -3)$$

Find the midpoint of the following:

1. Find the midpoint of the line segment with endpoints $(7, -2)$ and $(1, -4)$.
2. Find the midpoint of the line segment with endpoints $(2, -6)$ and $(5, 6)$.
3. Endpoints are $(-4, 4)$ and $(5, -2)$.
4. Endpoints are $(-1, 10)$ and $(-5, -3)$.
5. Find the midpoint of the line segment with endpoints $(-5, -2)$ and $(8, 2)$.

SLOPE

The steepness of a line on a coordinate plane is called its **slope**. Slope is measured as a ratio: the vertical (y) change between any two points on a line divided by the horizontal (x) change between the points. Vertical change is called **rise** (ryse) and horizontal change is called **run**, so we can say that slope equals “rise over run”:

Rise(vertical change) is the difference between the y -coordinates, and run (horizontal change) is the difference between the x -coordinates. So the **formula for the slope** is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The m is the symbol for slope and the formula is read as “Slope equals the change in y divided by the change in x .”

When comparing the rise (y) and run (x), if the rise is greater than the run, the line is steeper. If the run (x) is greater than the rise (y) the line is less steep.

When looking at the line from left to right, if the line **rises**, its **slope is positive**. If the line falls or goes down from left to right, its **slope is negative**.

EXAMPLE: Find the slope m of the line containing the points $(-5,3)$ and $(1, 4)$.

Answer: $m = \frac{4-3}{1-(-5)} = \frac{1}{6}$ the slope is positive.

Find the slope of the line between the two points and state whether it is positive or negative.

1. $(7, 3)$ and $(4, -2)$
2. $(9, -8)$ and $(-3, 2)$
3. $(-6, -4)$ and $(5, 2)$
4. $(0, -3)$ and $(-5, 0)$
5. $(8, -3)$ and $(0, 5)$

DISTANCE

To find the distance between two points on a coordinate graph, first determine if the points are on the same horizontal or vertical line.

If the points are on the same horizontal line, simply find the difference of the x coordinates. If the points are on the same vertical line, find the difference of the y coordinates. In each case, the difference is the distance between the two points. Remember that this distance must be a positive number. If the difference of the coordinates comes out negative, use the absolute value.

If the points are not on a vertical or horizontal line but rather on a diagonal line, you need to use a distance formula. This formula is based on the Pythagorean theorem. Given points (x_1, y_1) and (x_2, y_2) the distance d between them is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE: Find the distance d between $(3, 6)$ and $(-2, 5)$.

$$d = \sqrt{(3 - (-2))^2 + (6 - 5)^2}$$

$$d = \sqrt{(3 + 2)^2 + (1)^2}$$

$$d = \sqrt{5^2 + 1^2}$$

$$d = \sqrt{25 + 1}$$

$$d = \sqrt{26}$$

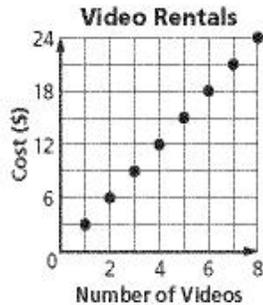
Find the distance, to the nearest tenth, between the points in each pair.

1. $(3, 8)$ and $(2, 14)$
2. $(-1, 4)$ and $(5, -3)$
3. $(6, 4)$ and $(12, 4)$
4. $(4, -2)$ and $(8, 8)$
5. $(-3, 7)$ and $(-3, 12)$

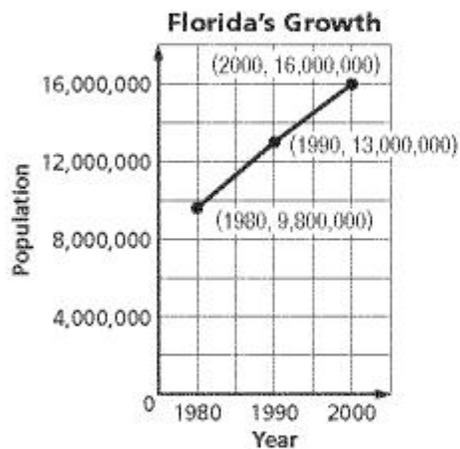
Fill in the missing values. Give the distance to the nearest tenth.

| | Points | Slope | Midpoint | Distance |
|-----|---------------------------|--------------|-----------------|-----------------|
| 1. | $(-5, 6)$ and $(1, -8)$ | | | |
| 2. | $(3, 8)$ and $(7, 2)$ | | | |
| 3. | $(-4, -9)$ and $(6, -2)$ | | | |
| 4. | $(-5, 4)$ and $(-5, 8)$ | | | |
| 5. | $(1, -2)$ and $(-9, -11)$ | | | |
| 6. | $(2, 5)$ and $(3, -4)$ | | | |
| 7. | $(8, -2)$ and $(-6, -2)$ | | | |
| 8. | $(-6, -10)$ and $(2, 12)$ | | | |
| 9. | $(-3, 9)$ and $(8, -2)$ | | | |
| 10. | $(4, 1)$ and $(-4, -3)$ | | | |

1. A video store customer graphed her rentals versus her cost on the coordinate plane. The slope between each pair of points is $\frac{3}{1}$. What does this represent?



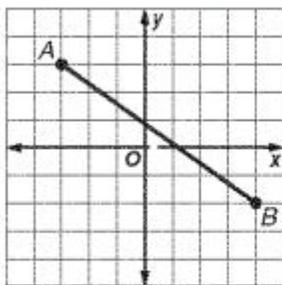
- a. She rented 3 videos per day. c. She rented 1 video every 3 days.
 b. The cost per rental is \$3 per video. d. The cost per rental is \$1 for 3 videos.
2. **THINK**
SOLVE
EXPLAIN The graph shows Florida's population growth between 1980 and 2000.



Part A Find the slope of each segment in each graph.

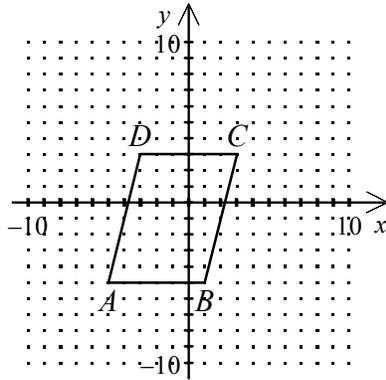
Part B What does the difference in the slopes indicate?

3. What is the midpoint of \overline{AB} ?



- a. (0, 1) c. (1, 0)
 b. (0.5, 0.5) d. (2, -0.5)

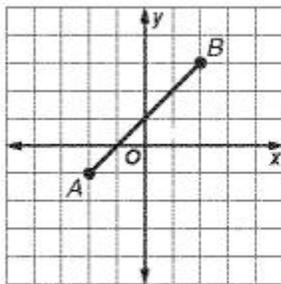
4. The diagonals of parallelogram $ABCD$ have a common midpoint.



Which of the following is the midpoint of the diagonals of $ABCD$?

- [A] $(-1, -1)$ [B] $(4, 4)$ [C] $(-1, 4)$ [D] $(4, -1)$

5. What is the length of \overline{AB} ?



- a. $2\sqrt{2}$ c. 5
b. $3\sqrt{2}$ d. $4\sqrt{2}$

6.

| |
|---------|
| THINK |
| SOLVE |
| EXPLAIN |
| |
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| |

The coordinates of the vertices of a quadrilateral are $A(0, -7)$, $B(3, -6)$, $C(15, -10)$, and $D(12, -11)$.

Part A How long is each side of the quadrilateral? Show your work.

Part B What are the slopes of each side of the quadrilateral? Show your work.

Part C What type of quadrilateral is it? Explain your reasoning.

7. The positions of two airplanes approaching an airport are plotted on a graph grid with the airport located at $(0, 0)$. The locations of the planes are given by the coordinates $(-5, 1)$ and $(8, -1)$. Each grid square is 1 mile wide. How far apart are the approaching airplanes? Round your answer to the nearest tenth of a mile.

8. The positions of two airplanes approaching an airport are plotted on a graph grid with the airport located at $(0, 0)$. The locations of the planes are given by the coordinates $(-8, 5)$ and $(-2, 2)$. Each grid square is 1 mile wide. How far apart are the approaching airplanes? Round your answer to the nearest tenth of a mile.